Eur. Phys. J. C 34, 237–244 (2004)
Digital Object Identifier (DOI) 10.1140/epjc/s2004-01602-3

THE EUROPEAN PHYSICAL JOURNAL C

$B\bar{B}$ mixing with the bulk fields in the Randall–Sundrum model

J.-P. Lee[a]

Department of Physics and IPAP, Yonsei University, Seoul, 120-749, Korea

Received: 15 October 2003 /
Published online: 18 February 2004 – © Springer-Verlag / Società Italiana di Fisica 2004

Abstract. We calculate $B\bar{B}$ mixing in the Randall–Sundrum bulk model. In this model, all the standard model fields except the Higgs field can reside in the bulk. Two suggestive models of "mixed" and "relaxed" scenarios are considered. We find that the enhancement of the loop function is 0.51% for the "relaxed" and 1.07% for the "mixed" scenario when the first four Kaluza-Klein modes are included, for a bulk fermion mass parameter $\nu = -0.3$.

1 Introduction

The idea of extra dimensions provides a very elegant explanation for the gauge hierarchy problem of the standard model (SM). Arkani-Hamed, Dimopoulos and Dvali (ADD) suggested that there exist n flat extra dimensions with factorizable geometry [1]. In this model, the enormous Planck scale $M_{\rm Pl}$ is induced by the largeness of the extra dimensional volume V_n, via $M_{\rm Pl}^2 = M_0^{n+2} V_n$. The new fundamental scale M_0 in $4+n$ dimensions can be set down around 1 TeV, which resolves the hierarchy problem. An alternative to ADD was soon proposed by Randall and Sundrum (RS) [2] where the gauge hierarchy is explained by an exponential warp factor from a 5-dimensional non-factorizable geometry. In this model, two branes are embedded at the boundaries of the AdS_5 slice with a single S_1/Z_2 orbifold extra dimension.

In the original model of ADD or RS, only gravity can propagate in the bulk. The effects of the bulk graviton appear in the 4D world as an infinite tower of Kaluza–Klein (KK) modes. Their phenomenological signatures at colliders are widely studied.

A natural extension as a next step is to put the SM fields into the bulk. In the universal extra dimension (UED) model by Appelquist, Cheng and Dobrescu (ACD), all the SM fields are allowed to live in the extra dimensions [3]. In the RS model, Goldberger and Wise tried to put the scalar fields into the bulk, which has been developed to the bulk-stabilizing modulus field, or radion [4]. They provided the open possibility that the SM matter fields might reside in the extra dimensions. Later the gauge bosons were placed in the bulk in [5]. But the electroweak precision data constrain the first KK state of the gauge boson so strongly that the typical scale on the TeV brane goes up to about 100 TeV. Apparently, this is not a good solution to the gauge hierarchy problem. The problem was alleviated by putting fermion fields in the bulk while confining the Higgs to the TeV brane [6]. An attempt of constructing the bulk fermion fields was already done in [7] to explain the neutrino masses and mixings.

Equipped with the RS-bulk SM fields, lots of works have been done to refine the model and accommodate it to the existing data. Here the fermion mass parameter $\nu \equiv m_\psi/k$ plays an important role, where m_ψ is the fermion bulk mass and $\mathcal{R}_5 = -20k^2$ is the 5-dimensional curvature [8,9]. When placing the fermion fields in the bulk, there might occur large contributions to the flavor changing neutral current (FCNC) or the SM ρ parameter. The simple assumptions of a universal bulk fermion mass and minimal flavor violation where the CKM paradigm governs the flavor mixing can be a solution to these potential problems.

However, when the Yukawa interaction is taken into account, non-negligible mixing in the top sector can give a large contribution to the ρ parameter. Even worse is that the shifted mass spectrum in the top quark KK modes does not guarantee the Glashow–Iliopoulos–Maiani (GIM) cancellation, which will cause a disastrous FCNC. As a more realistic model, Hewett–Petriello–Rizzo (HPR) proposed the so-called "mixed scenario" where the first two generations of fermions are placed in the bulk while the third is localized on the TeV brane [10]. While the HPR model reproduces the quark mass hierarchies m_c/m_t and m_s/m_b without spoiling the ρ parameter constraints, it seems unnatural to confine only one family into the wall; there still remains a potential danger of FCNC.

As an alternative, a "relaxed" model was suggested by Kim–Kim–Song (KKS) where the assumption of a universal bulk fermion mass is slightly modified, retaining all the fermions in the bulk [11]. In this approach, the $SU(2)$ singlet bottom quark field has a different bulk mass m'_ψ. They showed that introducing another parameter can be accommodated well to the electroweak precision data of $\Delta\rho \equiv \rho - \rho_{\rm SM}$ and $b \to s\gamma$.

[a] e-mail: jplee@phya.yonsei.ac.kr

In this paper, we analyze $B\bar{B}$ mixing based on the KKS. With the advent of the B-factory era, we are now entering the age of precision tests of flavor physics. The current world average of ΔM_B, which parameterizes $B\bar{B}$ mixing, is $\Delta M_B = 0.502 \pm 0.006\,\mathrm{ps}^{-1}$ [12], where the experimental error is very small. Relevant box diagrams were already studied in the ACD-UED model, resulting in a 17% enhancement of the loop function [13,14]. The present work will be a good comparison to the ACD-UED result. It is also interesting to compare the results from KKS and HPR with each other. If the fermion mass parameter $\nu = -0.3$ is chosen (which is phenomenologically viable in the literature), especially, mixing in the top sector is rather small; the GIM cancellation is incomplete and we expect some remnant. The diagonalization of the mass matrix can be done perturbatively. Naive thoughts lead to the idea that HPR will produce larger effects than KKS, yielding the amount of the excess of FCNC from the HPR that would appear.

This paper is organized as follows. The RS-bulk model is reviewed in the next section. We simply omit gravity in the setup. In Sect. 3, mixing in the top sector is considered. We follow the description of KKS [11], and do not consider possible gauge boson mixing for simplicity. The box diagrams for the $B\bar{B}$ mixing are calculated in Sect. 4. Section 5 contains the results and discussions. A summary is given in Sect. 6.

2 Setup of the model

In the RS model, one spatial dimension is compactified on a S_1/Z_2 orbifold of radius r_c with the non-factorizable geometry

$$\mathrm{d}s^2 = G_{MN}\mathrm{d}x^M\mathrm{d}x^N = \mathrm{e}^{-2\sigma(\phi)}\eta_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu + r_c^2\mathrm{d}\phi^2\ , \quad (1)$$

where the four-dimensional metric tensor is $\eta_{\mu\nu} = \mathrm{diag}(1,-1,-1,-1)$, and $\sigma(\phi) = kr_c|\phi|$, $0 \le |\phi| \le \pi$. The model parameter k is related to the 5-dimensional curvature $\mathcal{R}_5 = -20k^2$.

In the original RS model, only the graviton can propagate through the bulk with its KK modes. We do not consider the graviton KK modes here because they are irrelevant for $B\bar{B}$ mixing. We follow the model setup of KKS [11].

Non-abelian gauge fields $A_M^a(x,\phi)$ can reside in the bulk via the 5D action

$$S_A = -\frac{1}{4}\int \mathrm{d}^5x\sqrt{-G}G^{MK}G^{NL}F^a_{KL}F^a_{MN}\ , \quad (2)$$

where $F^a_{MN} = \partial_M A^a_N - \partial_N A^a_M - g_5\epsilon^{abc}A^b_M A^c_N$ $(a,b,c = 1,2,3)$. Choosing the gauge of $A_4(x,\phi) = 0$ and assuming the KK expansion of A_μ to be

$$A^a_\mu(x,\phi) = \sum_{n=0}^{\infty} A^{a(n)}_\mu(x)\frac{\chi^{(n)}_A(\phi)}{\sqrt{r_c}}\ , \quad (3)$$

we have a 4-dimensional effective action of massive KK gauge bosons as follows:

$$S_A = \int \mathrm{d}^4x\sum_{n=0}^{\infty}\left[-\frac{1}{4}\eta^{\mu\kappa}\eta^{\nu\lambda}F^{a(n)}_{\kappa\lambda}F^{a(n)}_{\mu\nu} - \frac{1}{2}M_A^{(n)2}\eta^{\mu\nu}A^{a(n)}_\mu A^{a(n)}_\nu\right]\ , \quad (4)$$

if the extra dimensional component $\chi^{(n)}_A(\phi)$ is the Bessel function

$$\chi^{(n)}_A = \frac{\mathrm{e}^{\sigma(\phi)}}{N^{(n)}_A}\left[J_1(z^{(n)}_A(\phi)) + \alpha^{(n)}_A Y_1(z^{(n)}_A(\phi))\right]\ . \quad (5)$$

Here

$$z^{(n)}_A(\phi) = \frac{M^{(n)}_A}{k}\mathrm{e}^{\sigma(\phi)}\ , \quad (6)$$

where $M^{(n)}_A$ is the mass of the nth KK mode of the gauge boson. The continuity of $\mathrm{d}\chi^{(n)}_A/\mathrm{d}\phi$ at $\phi = 0$ and $\phi = \pm\pi$ determines the mass spectrum and the coefficient $\alpha^{(n)}_A$,

$$\alpha^{(n)}_A = -\frac{J_1(M^{(n)}_A/k) + (M^{(n)}_A/k)J_1'(M^{(n)}_A/k)}{Y_1(M^{(n)}_A/k) + (M^{(n)}_A/k)Y_1'(M^{(n)}_A/k)} \quad (\text{at } \phi = 0)\ , \quad (7)$$

$$J_1(x^{(n)}_A) + x^{(n)}_A J_1'(x^{(n)}_A) + \alpha^{(n)}_A\left[Y_1(x^{(n)}_A) + x^{(n)}_A Y_1'(x^{(n)}_A)\right] = 0 \quad (\text{at } \phi = \pm\pi)\ , \quad (8)$$

where $x^{(n)}_A \equiv z^{(n)}_A(\phi = \pi) = (M^{(n)}_A/k)\mathrm{e}^{kr_c\pi}$. The normalization constant

$$N^{(n)}_A = \left(\frac{\mathrm{e}^{kr_c\pi}}{x^{(n)}_A\sqrt{kr_c}}\right) \times \sqrt{z^{(n)2}_A\left[J_1(z^{(n)}_A) + \alpha^{(n)}_A Y_1(z^{(n)}_A)\right]^2\Big|^{z^{(n)}_A(\phi=\pi)}_{z^{(n)}_A(\phi=0)}}\ , \quad (9)$$

is obtained by the orthonormality condition

$$\int_{-\pi}^{\pi}\mathrm{d}\phi\,\chi^{(n)}_A\chi^{(m)}_A = \delta^{mn}\ . \quad (10)$$

Now consider the bulk fermion with arbitrary Dirac bulk mass. A Dirac fermion field Ψ with bulk mass m_ψ is described by the 5-dimensional action [7]

$$S_F = \int \mathrm{d}^4x\int \mathrm{d}\phi\sqrt{-G}\left\{E^A_{\underline{A}}\left[\frac{\mathrm{i}}{2}\bar{\Psi}\gamma^{\underline{A}}(D_A - \overleftarrow{D}_A)\Psi\right] - m_\psi\mathrm{sgn}(\phi)\bar{\Psi}\Psi\right\}\ , \quad (11)$$

where D_A is the covariant derivative, $\gamma^{\underline{A}} = (\gamma^\mu, \mathrm{i}\gamma_5)$, and the inverse vielbein $E^A_{\underline{A}} = \mathrm{diag}(\mathrm{e}^\sigma, \mathrm{e}^\sigma, \mathrm{e}^\sigma, \mathrm{e}^\sigma, 1/r_c)$. The underlined uppercase Roman indices describe objects in the tangent frame. The possible spin connection $\omega_{\underline{BC}A}$ is omitted because it vanishes when the Hermitian conjugate is included.

Integration by parts leads to

$$S_F = \int d^4x \int d\phi r_c \Big\{ e^{-3\sigma} \left(\bar{\Psi}_L i\partial\!\!\!/ \Psi_L + \bar{\Psi}_R i\partial\!\!\!/ \Psi_R \right) - \frac{1}{2r_c} \left[\bar{\Psi}_L (e^{-4\sigma}\partial_\phi + \partial_\phi e^{-4\phi}) \Psi_R - \bar{\Psi}_R (e^{-4\sigma}\partial_\phi + \partial_\phi e^{-4\phi}) \Psi_L \right] - e^{-4\sigma} m_\psi \mathrm{sgn}(\phi) (\bar{\Psi}_L \Psi_R + \bar{\Psi}_R \Psi_L) \Big\} , \quad (12)$$

where the periodic boundary conditions of $\Psi_{L,R}(x,\pi) = \Psi_{L,R}(x,-\pi)$ are imposed. Expanding Ψ as

$$\Psi_{L,R}(x,\phi) = \sum_{n=0}^{\infty} \psi_{L,R}^{(n)} \frac{e^{2\sigma(\phi)}}{\sqrt{r_c}} \hat{f}_{L,R}^{(n)}(\phi) , \quad (13)$$

and requiring that

$$\int_{-\pi}^{\pi} d\phi e^{\sigma(\phi)} \hat{f}_L^{(m)*}(\phi) \hat{f}_L^{(n)}(\phi) = \int_{-\pi}^{\pi} d\phi e^{\sigma(\phi)} \hat{f}_R^{(m)*}(\phi) \hat{f}_R^{(n)}(\phi) = \delta^{mn} , \quad (14)$$

$$\left(\pm \frac{1}{r_c} \partial_\phi - m_\psi \right) \hat{f}_{L,R}^{(n)}(\phi) = -M_f^{(n)} e^{\sigma} \hat{f}_{R,L}^{(n)}(\phi), \quad (15)$$

we have the effective action for the massive Dirac fermions

$$S_F = \sum_{n=0}^{\infty} \int d^4x \left[\bar{\psi}^{(n)}(x) i\partial\!\!\!/ \psi^{(n)}(x) - M_f^{(n)} \bar{\psi}^{(n)}(x) \psi^{(n)}(x) \right] . \quad (16)$$

The Z_2-symmetry of S_F in (11) requires that $\bar{\Psi}\Psi = \bar{\Psi}_L \Psi_R + \bar{\Psi}_R \Psi_L$ be Z_2-odd. This requirement is satisfied if we impose the opposite Z_2-parity on $\hat{f}_{L,R}^{(n)}$. We fix $\hat{f}_{L(R)}^{(n)}$ to be Z_2-even (-odd). Introducing $\nu \equiv m_\psi / k$ of order 1, the solutions are, for $n \neq 0$,

$$\hat{f}_L^{(n)} \equiv \chi^{(n)}(\phi) = \frac{e^{\sigma/2}}{N_\chi^{(n)}} \left[J_{1/2-\nu}(z^{(n)}) + \beta_\chi^{(n)} Y_{1/2-\nu}(z^{(n)}) \right] , \quad (17)$$

$$\hat{f}_R^{(n)} \equiv \tau^{(n)}(\phi) = \frac{e^{\sigma/2}}{N_\tau^{(n)}} \left[J_{1/2+\nu}(z^{(n)}) + \beta_\tau^{(n)} Y_{1/2+\nu}(z^{(n)}) \right] , \quad (18)$$

where $z_f^{(n)}(\phi) \equiv (M_f^{(n)}/k) e^{\sigma(\phi)}$, and for $n = 0$,

$$\chi^{(0)}(\phi) = \frac{e^{\nu\sigma(\phi)}}{N_\chi^{(0)}} , \quad \tau^{(0)}(\phi) = 0 . \quad (19)$$

The mass spectrum and the coefficients $\beta_{\chi,\tau}^{(n)}$ as well as the normalization constants $N_{\chi,\tau}^{(n)}$ are determined by the boundary conditions

$$0 = \left(\frac{d}{d\phi} - m_\psi r_c \right) \chi^{(n)}|_{\phi=0,\pi} = \tau^{(n)}|_{\phi=0,\pi} . \quad (20)$$

Explicitly,

$$\beta_\chi^{(n)} = - \frac{J_{-(1/2+\nu)}(M_f^{(n)}/k)}{Y_{-(1/2+\nu)}(M_f^{(n)}/k)} , \quad (21)$$

from the boundary conditions at $\phi = 0$, and

$$J_{-(1/2+\nu)}(x_f^{(n)}) + \beta_\chi^{(n)} Y_{-(1/2+\nu)}(x_f^{(n)}) = 0 , \quad (22)$$

at $\phi = \pi$, where $x_f^{(n)} \equiv z_f^{(n)}(\phi = \pi)$. Similar expressions can be given for the $\tau^{(n)}(\phi)$ sector, and the left- and right-handed excitation masses are degenerate to $M_f^{(n)}$ for a given n. The normalization constants are

$$N_{\chi,\tau}^{(n)} = \left(\frac{e^{kr_c\pi}}{x_f^{(n)} \sqrt{kr_c}} \right) \times \sqrt{ z_f^{(n)2} \left[J_{1/2\mp\nu}(z_f^{(n)}) + \beta_{\chi,\tau}^{(n)} Y_{1/2\mp\nu}(z_f^{(n)}) \right]^2 \Big|_{z_f^{(n)}(\phi=0)}^{z_f^{(n)}(\phi=\pi)} } . \quad (23)$$

Some difficulties arise when the fermions are included in the bulk because the fermion field contents must be doubled. In our setup of (12), a fermion which belongs to a specific representation of a gauge group should be vector-like, possessing both left- and right-handed chiralities. For each generation, we have an $SU(2)$ doublet $Q = (q_u, q_d)^T$ and two $SU(2)$ singlets u and d where both chiralities are allowed for all of them. Explicitly,

$$Q(x,\phi) = Q_L + Q_R = \sum_n \frac{e^{2\sigma(\phi)}}{\sqrt{r_c}} \left[Q_L^{(n)}(x) \chi^{(n)}(\phi) + Q_R^{(n)}(x) \tau^{(n)}(\phi) \right] , \quad (24)$$

$$u(x,\phi) = u_L + u_R = \sum_n \frac{e^{2\sigma(\phi)}}{\sqrt{r_c}} \left[u_L^{(n)}(x) \tau^{(n)}(\phi) + u_R^{(n)}(x) \chi^{(n)}(\phi) \right] , \quad (25)$$

where we have assigned $\chi^{(n)}$ to the left-handed $SU(2)$ doublet and the right-handed $SU(2)$ singlet in order to accommodate the SM fermion fields. The $SU(2)$ singlet down part $d(x,\phi)$ has the same KK decomposition as $u(x,\phi)$.

With these bulk-SM fields, the charged current interactions look like

$$S_{ffW} = \int d^5x \sqrt{-G} e^{\sigma} \frac{g_5}{\sqrt{2}} \left[\bar{q}_u W\!\!\!\!/^{+} q_d + \text{h.c.} \right] = \int d^4x \frac{g}{\sqrt{2}} \sum_{l=0}^{\infty} \Bigg[\sum_{n,m=0}^{\infty} \bar{q}_{uL}^{(n)} W\!\!\!\!/^{+(l)} q_{dL}^{(m)} C_{nml}^{ffW} + \sum_{n,m=1}^{\infty} \bar{q}_{uR}^{(n)} W\!\!\!\!/^{+(l)} q_{dR}^{(m)} \left(\sqrt{2\pi} \int_{-\pi}^{\pi} d\phi \, e^{\sigma} \tau^{(n)} \tau^{(m)} \chi_A^{(l)} \right) \Bigg] + \text{h.c.} , \quad (26)$$

where $g = g_5/\sqrt{2\pi r_c}$ and the coupling constant C_{nml}^{ffW} is

$$C_{nml}^{ffW} = \sqrt{2\pi} \int_{-\pi}^{\pi} d\phi \, e^{\sigma} \chi^{(n)}(\phi) \chi^{(m)}(\phi) \chi_A^{(l)}(\phi) . \quad (27)$$

3 Mixing in the top sector

For the bulk fermions, there are two sources of the physical fermion mass spectrum. One is the KK mass which is proportional to $\sim \bar{q}_{u\mathrm{L}} q_{u\mathrm{R}}$ or $\sim \bar{u}_\mathrm{L} u_\mathrm{R}$. The other is the Yukawa interaction which relates the $SU(2)$ doublet with the singlet, $\sim \bar{q}_{u\mathrm{L}} u_\mathrm{R}$, as in the SM. This difference results in mixing among the fermion KK modes. If the quark masses were sufficiently smaller than the KK mass scale, these mixing effects might be negligible. In this case, all the fermion KK mass spectrum would be almost degenerate assuming that the model is minimal with a single parameter m_ψ for the universal bulk fermion mass. The exact degeneracy between the KK masses for the $T_3 = \pm 1/2$ fermions is responsible for the vanishing contribution to $\Delta\rho$. Furthermore, we can expect the GIM cancellation at each level of KK modes because all the up- (and down-) type quarks are degenerate.

The presence of the top quark, which is very heavy, makes the mixing non-negligible. Consequently, degenerate top and bottom KK masses are shifted. They can cause a large $\Delta\rho$, because the quantum correction is proportional to the squared mass difference between up- and down-type quarks. Adding higher KK modes makes the problem worse.

As mentioned in the Introduction, HRP proposed the "mixed" scenario, where the third-generation fermions are confined to the TeV brane while the other two can propagate in the bulk. Instead, we adopt the KKS model of [11] where the $SU(2)$ singlet bottom quark field has a different bulk fermion mass m'_ψ. The model is recapitulated in the 5D action for the third-generation quarks

$$S = \int \mathrm{d}^4 x \int \mathrm{d}\phi \sqrt{-G} \times \left[E_a^A (\mathrm{i}\bar{Q}\gamma^a \mathcal{D}_A Q + \mathrm{i}\bar{t}\gamma^a \mathcal{D}_A t + \mathrm{i}\bar{b}\gamma^a \mathcal{D}_A b) - \mathrm{sgn}(\phi)(m_\psi \{\bar{Q}Q + \bar{t}t\} + m'_\psi \bar{b}b) \right] . \quad (28)$$

Expanding the fermion fields with the mode functions and integrating over ϕ yield the bulk fermion KK masses:

$$\mathcal{L} = -\sum_{n=1}^{\infty} k_\mathrm{EW} \left[x_f^{(n)}(\nu) \left(\bar{q}_{t\mathrm{L}}^{(n)} q_{t\mathrm{R}}^{(n)} + \bar{q}_{b\mathrm{L}}^{(n)} q_{b\mathrm{R}}^{(n)} + \bar{t}_\mathrm{L}^{(n)} t_\mathrm{R}^{(n)} \right) + x_f^{(n)}(\nu') \bar{b}_\mathrm{L}^{(n)} b_\mathrm{R}^{(n)} \right] + \mathrm{h.c.} , \quad (29)$$

where $k_\mathrm{EW} = k\epsilon \equiv k\mathrm{e}^{-kr_c\pi}$, and $\nu' \equiv m'_\psi / k$.

Now consider the Yukawa interactions associated with the Higgs boson. The ordinary Higgs mechanism works here so that the SM particles acquire masses. It has been argued, however, that if there were bulk Higgs fields, the hierarchy problem remains unsolved [6] or the observed W and Z mass relations cannot be reproduced [9]. We assume here that the Higgs field is confined to the TeV brane. Then the 5D action for the Yukawa interactions is

$$S_{ffH} = -\int \mathrm{d}^5 x \sqrt{-G} \times \left[\frac{\lambda_5^b}{k} \bar{Q}(x,\phi) \cdot H(x) b(x,\phi) + \frac{\lambda_5^t}{k} \epsilon^{ab} \bar{Q}(x,\phi)_a \cdot H(x)_b t(x,\phi) + \mathrm{h.c.} \right] \times \delta(\phi - \pi) , \quad (30)$$

where $\lambda_5^{b,t}$ is the 5D Yukawa coupling. After the spontaneous symmetry breaking, the Higgs field shifts $H^0 \to v_5 + H'^0$, and the 4D effective Lagrangian becomes

$$\mathcal{L}_\mathrm{eff} = \frac{\lambda_t v}{\sqrt{2}} \left(\bar{q}_{t\mathrm{L}}^{(0)} + \hat{\chi}_1 \bar{q}_{t\mathrm{L}}^{(1)} + \cdots \right) \left(t_\mathrm{R}^{(0)} + \hat{\chi}_1 t_\mathrm{R}^{(1)} + \cdots \right) + \frac{\lambda_b v}{\sqrt{2}} \left(\bar{q}_{b\mathrm{L}}^{(0)} + \hat{\chi}'_1 \bar{q}_{b\mathrm{L}}^{(1)} + \cdots \right) \left(b_\mathrm{R}^{(0)} + \hat{\chi}'_1 b_\mathrm{R}^{(1)} + \cdots \right) , \quad (31)$$

where $\lambda_{t,b} = \lambda_5^{t,b}(1+2\nu)/2(1-\epsilon^{1+2\nu})$, $v = \epsilon v_5$, $\hat{\chi}_n \equiv \chi^{(n)}(\pi,\nu)/\chi^{(0)}(\pi,\nu)$, and $\hat{\chi}'_n$ are associated with ν'.

From (29) and (31), the physical mass term for the top sector is

$$\mathcal{L}_\mathrm{mass}^t = -\left(\bar{t}_\mathrm{R}^{(0)}\ \bar{t}_\mathrm{R}^{(1)} \cdots \,\middle|\, \bar{q}_{t\mathrm{R}}^{(1)} \cdots \right) \mathcal{M}_t \begin{pmatrix} q_{t\mathrm{L}}^{(0)} \\ q_{t\mathrm{L}}^{(1)} \\ \vdots \\ - \\ t_\mathrm{L}^{(1)} \\ \vdots \end{pmatrix} . \quad (32)$$

The bottom sector shows very similar mass terms. Let the number of the KK states be n_∞ which is, in principle, infinite. Then the $(2n_\infty + 1) \times (2n_\infty + 1)$ matrix $\mathcal{M}_t$ has the form

$$\mathcal{M}_t = \begin{pmatrix} \mathcal{M}_Y^t & \mathcal{M}_{KK}^t \\ \mathcal{M}_{KK}^{q_t} & 0 \end{pmatrix} , \quad (33)$$

where the $(n_\infty + 1) \times (n_\infty + 1)$ matrix $\mathcal{M}_Y^t$ originates from the Yukawa mass term and the $(n_\infty + 1) \times n_\infty$ matrix $\mathcal{M}_{KK}$ from the KK masses. They are given by

$$\mathcal{M}_Y^t = m_{t,0} \begin{pmatrix} 1 & \hat{\chi}_1 & \hat{\chi}_2 & \cdots \\ \hat{\chi}_1 & \hat{\chi}_1^2 & \hat{\chi}_1\hat{\chi}_2 & \cdots \\ \hat{\chi}_2 & \hat{\chi}_2\hat{\chi}_1 & \hat{\chi}_2^2 & \cdots \\ \vdots & \vdots & \vdots & \end{pmatrix} , \quad (34)$$

$$\mathcal{M}_{KK}^{q_t} = k_\mathrm{EW} \begin{pmatrix} 0 & x_f^{(1)} & 0 & \cdots \\ 0 & 0 & x_f^{(2)} & \cdots \\ 0 & 0 & 0 & \cdots \\ \vdots & \vdots & \vdots & \end{pmatrix} = \mathcal{M}_{KK}^t , \quad (35)$$

where $m_{t,0} = \lambda_t v / \sqrt{2}$.

Now the physical mass eigenstates $u_{\rm L}^{\prime(n)}$ are obtained by diagonalizing (33) through an orthogonal matrix $\mathcal{N}$:

$$\begin{pmatrix} u_{\rm L}^{\prime(0)} \\ u_{\rm L}^{\prime(1)} \\ u_{\rm L}^{\prime(2)} \\ \vdots \end{pmatrix} = \mathcal{N} \begin{pmatrix} q_{u{\rm L}}^{(0)} \\ q_{u{\rm L}}^{(1)} \\ \vdots \\ - \\ u_{\rm L}^{(1)} \\ \vdots \end{pmatrix} . \qquad (36)$$

For example, $q_{u{\rm L}}^{(n)}$ can be written in terms of the KK mass eigenstates $u_{\rm L}^{\prime(\underline{j})}$:

$$q_{u{\rm L}}^{(n)} = \sum_{\underline{j}=0}^{2n_\infty} \mathcal{N}_{(\underline{j},n)} u_{\rm L}^{\prime(\underline{j})} . \qquad (37)$$

Hereafter, the underlined index runs from zero to $2n_\infty$. In terms of mass eigenstates, the Wtb-vertex in 4D becomes

$$\mathcal{L} = \frac{g}{\sqrt{2}} V_{tb} \left(\sum_{m=0}^{n_\infty} C_{0ml}^{btW} \mathcal{N}_{(\underline{j},m)} \right) \bar{b}_{\rm L}^{(0)} \gamma^\mu t_{\rm L}^{\prime(\underline{j})} W_\mu^{(l)} + \text{h.c.} \qquad (38)$$

For light quarks ($m_{q,0} = 0$), $\mathcal{M}_q$ can be analytically diagonalized to $\mathcal{M}_q \to \text{diag}(m_q^{(0)}, M_1^{(0)}, M_2^{(0)}, \cdots)$, where

$$m_q^{(0)} = 0 , \quad M_n^{(0)} = M_{n_\infty+n}^{(0)} = x_f^{(n)} k_{\rm EW} . \qquad (39)$$

The corresponding orthonormal matrix $\mathcal{N}^{(0)}$ is

$$\mathcal{N}_{(0,0)}^{(0)} = 1 , \quad \mathcal{N}_{(0,\underline{n})}^{(0)} = \mathcal{N}_{(\underline{n},0)}^{(0)} = 0 ,$$
$$\mathcal{N}_{(n,m)}^{(0)} = -\mathcal{N}_{(n_\infty+n,m)}^{(0)} = \frac{\delta_{nm}}{\sqrt{2}} . \qquad (40)$$

On the other hand, the diagonalization of $\mathcal{M}_t$ of the top sector is non-trivial since $m_{t,0}$ is heavy. However, an approximate calculation is possible unless the $\hat{\chi}_n^2$ are much larger than unity, with a small parameter $m_{t,0}/k_{\rm EW}$. It is shown in [11] that a perturbative diagonalization can be done for $\nu \gtrsim -0.3$. Up to the leading order of the expansion parameter $\delta \equiv m_{t,0}/k_{\rm EW}$, the mass eigenvalues are

$$m_q = m_0 , \quad M_n = k_{\rm EW} \left(x_f^{(n)} + \frac{\hat{\chi}_n^2}{2} \delta \right) ,$$
$$M_{n_\infty+n} = k_{\rm EW} \left(x_f^{(n)} - \frac{\hat{\chi}_n^2}{2} \delta \right) , \qquad (41)$$

and the orthonormal matrix $\mathcal{N}$ is

$$\mathcal{N}_{(\underline{n},\underline{m})} \equiv \mathcal{N}_{(\underline{n},\underline{m})}^{(0)} + \mathcal{N}_{(\underline{n},\underline{m})}^{(1)} \frac{\delta}{\sqrt{2}} , \qquad (42)$$

where

$$\mathcal{N}_{(0,0)}^{(1)} \simeq 0 , \quad \mathcal{N}_{(0,n)}^{(1)} \simeq 0 ,$$
$$\mathcal{N}_{(n,0)}^{(1)} \simeq \mathcal{N}_{(n_\infty+n,0)}^{(1)} \simeq \frac{\hat{\chi}_n^2}{x_f^{(n)}} ,$$
$$\mathcal{N}_{(n,n)}^{(1)} \simeq \mathcal{N}_{(n_\infty+n,n)}^{(1)} \simeq \frac{\hat{\chi}_n^2}{4 x_f^{(n)}} ,$$
$$\mathcal{N}_{(n,m)}^{(1)} \simeq \mathcal{N}_{(n_\infty+n,m)}^{(1)} \simeq \frac{x_f^{(n)} \hat{\chi}_n \hat{\chi}_m}{\sqrt{2}({x_f^{(n)}}^2 - {x_f^{(m)}}^2)} \quad (\text{for } n \neq m) . \qquad (43)$$

4 $B\bar{B}$ mixing and the box diagrams

$B\bar{B}$ mixing is parametrized by the physical mass difference

$$\Delta M_B = \frac{1}{M_B} |\langle \bar{B}^0 | \mathcal{H}_{\rm eff}(\Delta B = 2) | B^0 \rangle |$$
$$= \frac{G_{\rm F}^2 M_W^2}{6\pi^2} |V_{tb}^* V_{td}|^2 \hat{B}_B f_B^2 M_B \eta_B S_0(x_t) , \qquad (44)$$

where M_B (M_W) is the B (W) mass, η_B the QCD corrections, and $\hat{B}_B$ and f_B are the bag parameter and decay constant, respectively. The loop function $S_0(x_t)$ which recapitulates the box diagrams is

$$S_0(x_t) = \frac{4x_t - 11x_t + x_t^3}{4(1-x_t)^2} - \frac{3x_t^3 \ln x_t}{2(1-x_t)^3} , \qquad (45)$$

in the SM, with $x_t \equiv (m_t/M_W)^2$.

In the RS-bulk model, bulk fermions and gauge bosons contribute to the box diagrams, as shown in Fig. 1. Since the external fermions are the zero modes, the relevant vertex factor is C_{m0n}^{ffW} multiplied by the mass-diagonalizing orthonormal matrix $\mathcal{N}$.

Introducing the effective couplings

$$T^{ij} \equiv \left(\mathcal{N} \cdot C^{b(d)tW} \right)^{ij} , \qquad (46)$$
$$Q^{ij} \equiv \left(\mathcal{N} \cdot C^{b(d)qW} \right)^{ij} , \qquad (47)$$

the loop function has the form of

$$S(m,n;i,j)$$
$$= M_W^2 \sum \Bigg[\left(1 + \frac{m_{t^{(i)}}^2 m_{t^{(j)}}^2}{4 {M_A^{(m)}}^2 {M_A^{(n)}}^2} \right)$$
$$\times T^{im} T^{in} T^{jn} T^{jm} I(W^{(m)} W^{(n)} t^{(i)} t^{(j)})$$
$$+ \left(1 + \frac{m_{q^{(i)}}^2 m_{q^{(j)}}^2}{4 {M_A^{(m)}}^2 {M_A^{(n)}}^2} \right)$$
$$\times Q^{im} Q^{in} Q^{jn} Q^{jm} I(W^{(m)} W^{(n)} q^{(i)} q^{(j)})$$
$$- \left(2 + \frac{m_{q^{(i)}}^2 m_{t^{(j)}}^2 + m_{t^{(i)}}^2 m_{q^{(j)}}^2}{4 {M_A^{(m)}}^2 {M_A^{(n)}}^2} \right)$$
$$\times Q^{im} Q^{in} T^{jn} T^{jm} I(W^{(m)} W^{(n)} q^{(i)} t^{(j)})$$

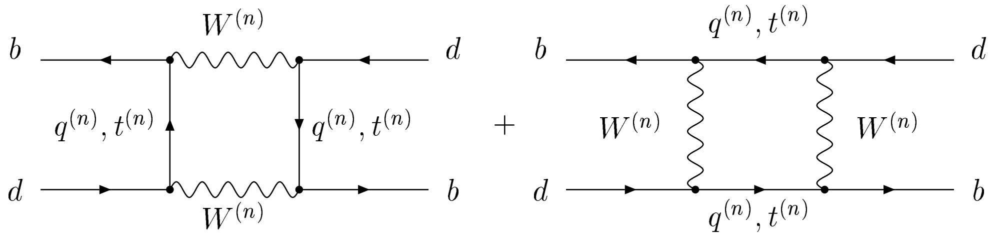

Fig. 1. Bulk gauge bosons and bulk fermions contribute to the box diagrams

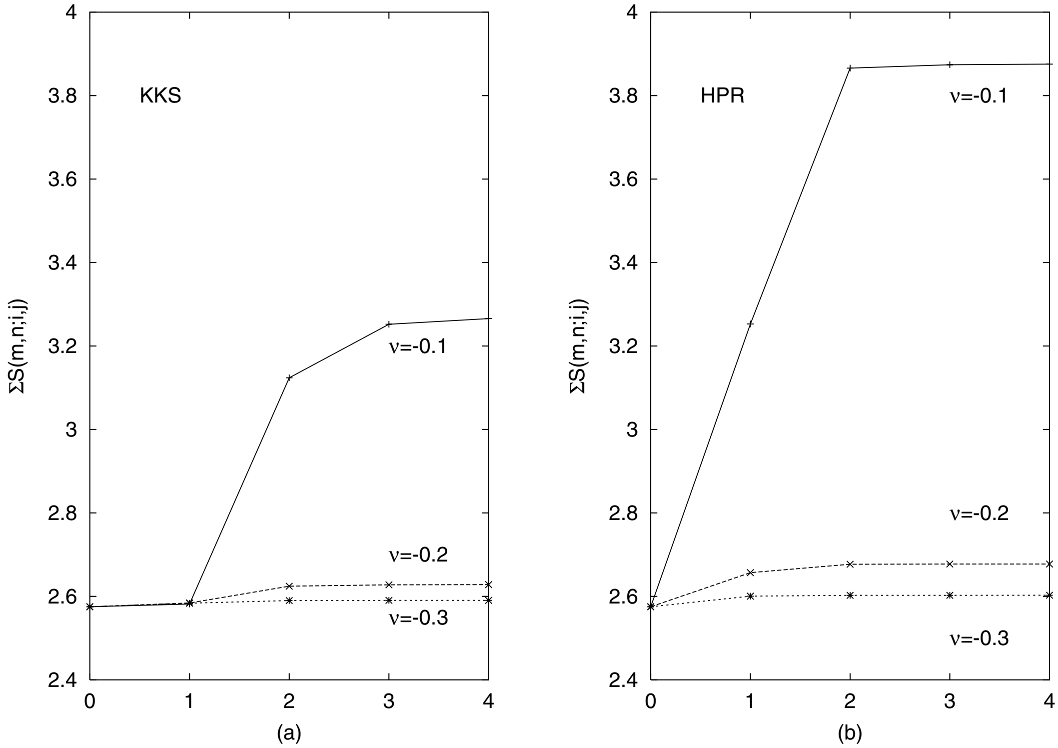

Fig. 2. Enhancements of the loop function $\sum S(m,n;i,j)$ for **a** KKS and **b** HPR for various ν. The x-axis is the number of KK modes included

$$
\begin{aligned}
&+\left(\frac{1}{M_A^{(m)^2}}+\frac{1}{M_A^{(n)^2}}\right)\\
&\quad\times\Big\{-m^2_{t^{(i)}}m^2_{t^{(j)}}T^{im}T^{in}T^{jn}T^{jm}I(W^{(n)}\phi^{(m)}t^{(i)}t^{(j)})\\
&\quad -m^2_{q^{(i)}}m^2_{q^{(j)}}Q^{im}Q^{in}Q^{jn}Q^{jm}I(W^{(n)}\phi^{(m)}q^{(i)}q^{(j)})\\
&\quad +(m^2_{q^{(i)}}m^2_{t^{(j)}}+m^2_{t^{(i)}}m^2_{q^{(j)}})\\
&\qquad\times Q^{im}Q^{in}T^{jn}T^{jm}I(W^{(n)}\phi^{(m)}q^{(i)}t^{(j)})\Big\}\Big]\ . \qquad (48)
\end{aligned}
$$

Here we separate the longitudinal part of $W^{(n)}$, denoted by $\phi^{(n)}$, for calculational convenience, and

$$
I(W^{(m)}W^{(n)}q^{(i)}q^{(j)})=\int_0^1 \mathrm{d}x\mathrm{d}y \qquad (49)
$$

$$
\times\left[\frac{M^2_{mn}+m^2_{q^{(i)}q^{(j)}}}{(M^2_{mn}-m^2_{q^{(i)}q^{(j)}})^2}+\frac{2m^2_{q^{(i)}q^{(j)}}M^2_{mn}}{(M^2_{mn}-m^2_{q^{(i)}q^{(j)}})^3}\ln\frac{m^2_{q^{(i)}q^{(j)}}}{M^2_{mn}}\right],
$$

$$
I(W^{(n)}\phi^{(m)}q^{(i)}q^{(j)})=\int_0^1 \mathrm{d}x\mathrm{d}y \qquad (50)
$$

$$
\times\left[\frac{2}{(M^2_{mn}-m^2_{q^{(i)}q^{(j)}})^2}+\frac{M^2_{mn}+m^2_{q^{(i)}q^{(j)}}}{(M^2_{mn}-m^2_{q^{(i)}q^{(j)}})^3}\ln\frac{m^2_{q^{(i)}q^{(j)}}}{M^2_{mn}}\right],
$$

where

$$
\begin{aligned}
m^2_{q^{(i)}q^{(j)}} &\equiv x m^2_{q^{(i)}}+(1-x)m^2_{q^{(j)}}\ ,\\
M^2_{mn} &\equiv y M_A^{(m)^2}+(1-y)M_A^{(n)^2}\ . \qquad (51)
\end{aligned}
$$

5 Results and discussions

For the numerical results, we used $kr_c = 11.5$, $k_{\mathrm{EW}} = 1\,\mathrm{TeV}$, and $\nu = -0.3$. The choice of $\nu = -0.3$ allows us to use (42) and (43); for $\nu \le -0.4$, only the numerical diagonalization of the mass matrix is reliable. The question may arise whether this choice would spoil the $\Delta\rho$ accommodation. This, however, would not be the case, because we

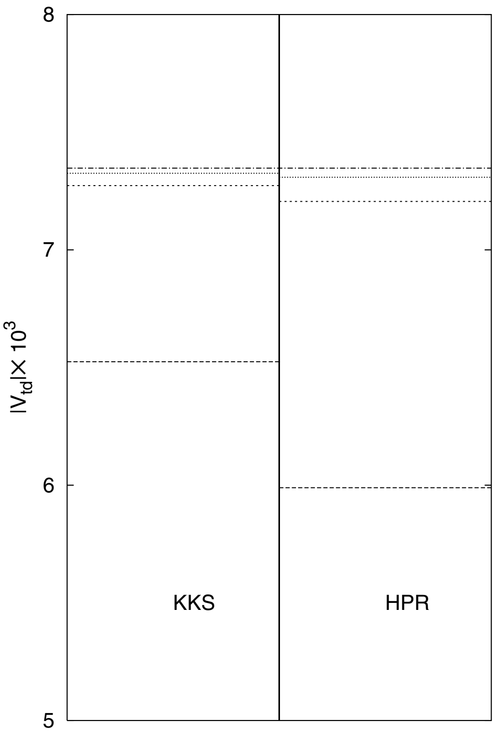

Fig. 3. Results for $|V_{td}|$ from the loop functions for KKS and HPR. The lines correspond to $\nu = -0.1, -0.2, -0.3$ from bottom to top, respectively. The top line is the SM value

Table 1. Enhancements of the loop fuction $\sum S(m, n; i, j)$ in % for KKS and HPR for different values of ν

	$\nu = -0.3$	$\nu = -0.2$	$\nu = -0.1$
KKS	0.51	2.06	26.8
HPR	1.07	3.98	50.5

Table 2. Values of C^{btW}_{011} and $x_f^{(1)}$ for various ν

	$\nu = -0.3$	$\nu = -0.2$	$\nu = -0.1$
C^{btW}_{011}	2.91	4.20	7.33
$x_f^{(1)} k_{\rm EW}$ (TeV)	2.71	2.85	3.00

have another adjustable parameter ν' for the b-sector in the KKS model. Even in the case of $(\nu, \nu') = (-0.3, -0.6)$ the mass differences between top and bottom KK modes are much smaller than 1 TeV[1]. We also give the results for $\nu = -0.2, \ -0.1$ for comparison.

In this analysis, we include the KK modes only up to the $n = 4$ gauge boson (which corresponds to the $n = 2$ fermion because a bulk fermion has double field contents) for simplicity. For comparison, the HPR contribution to the box diagrams is also given. Numerical results are summarized in Table 1 and Figs. 2 and 3. The main results of the analysis are as follows.

First, we have the loop function enhancement 0.51% (1.07%) for $\nu = -0.3$ in KKS (HPR) up to $n = 4$, as shown in Table 1 and Fig. 2. The numbers can be compared to the ACD-UED result where the increase amounts to 17% [14]. An increase of the loop function implies a smaller value of $|V_{td}|$, because $\Delta M_B = 0.502 \pm 0.006\,\mathrm{ps}^{-1} \sim |V_{td}|^2 \sum S(m, n; i, j)$ is now well measured [12]. Figure 3 shows our results for $|V_{td}|$. We used $\sqrt{\hat{B}_B} f_B = 235\,\mathrm{MeV}$, $\eta_B = 0.55$. The QCD correction factor η_B has a small error ± 0.01 compared to the hadronic uncertainty [14, 15]:

$$\sqrt{\hat{B}_B} f_B = (235^{+33}_{-41})\,\mathrm{MeV}\ . \tag{52}$$

Since $B_s\bar{B}_s$ mixing has the same loop function as $B\bar{B}$ mixing, an increase in $\sum S(m, n; i, j)$ predicts an equal amount of larger ΔM_{B_s}. The current bound of it is $\Delta M_{B_s} > 15\,\mathrm{ps}^{-1}$ [12].

In order to strongly constrain the model parameters, or even rule them out, measurements of $\sin 2\beta$, e.g., would not be crucial. Rather, determining the CKM angle γ as well as reducing the hadronic uncertainty significantly will severely restrict the parameter space. For example, in the ACD-UED model, [13] argued that a larger value of $\sqrt{\hat{B}_B} f_B$ above 222 MeV with one-third reduced error will rule out the UED model or push up the compactification scale up to the multi-TeV region. An analytic study is much easier in the ACD-UED model compared to the RS-type one because the mass spectrum and mode functions have a simpler structure in the UED model [3]. In the RS-bulk models, on the other hand, the n-dependence of the mass spectra, e.g., is not explicit.

Second, the HPR gives larger values than KKS for each n. The reason is as follows. If all the fermions were in the bulk and no mixing occurred, there would be GIM cancellation at every nth excitation. In case of KKS, the cancellation is incomplete because there is a small mixing (when $\nu \gtrsim -0.3$) in the top sector. In HPR, however, the third generation resides in the 3-brane; only the first and second generations have their KK excitations in the bulk. There is no compensating contribution from the third generation KK modes, which results in a larger loop function. This feature was already pointed out in [11].

The GIM cancellation is advantageous from the viewpoint of convergence. There is, of course, no way to warrant the finiteness of adding up an infinite KK tower by checking just a few excitations. In the ACD-UED, it is argued that the GIM mechanism improves the convergence of the loop function in $B\bar{B}$ mixing [14]. Also, it is quite encouraging that the RS-bulk SM effects of KKS on $b \to s\gamma$ are satisfactorily converging as we add higher KK modes [11].

Third, the ν-dependence is shown in Fig. 2 and Table 1. The KK mode contributions to the box diagrams get larger as ν grows. The reason is that both the coupling and the bulk fermion mass become larger when ν goes from -3 to -1. This pattern has already been studied in previous works; see Fig. 1 of [9] and [10]. We have, for example, the bulk fermion masses $x_f^{(1)} k_{\rm EW} = 2.71,\ 2.85,\ 3.00$ (TeV) for $\nu = -0.3, \ -0.2, \ -0.1$, respectively. It has already been suggested that a plausible range of ν is $-0.8 \lesssim \nu \lesssim -0.3$ in the literature [9, 10]. If $\nu \gtrsim -0.3$, constraints from the

[1] See, for example, Fig. 4 of [11].

electroweak precision data make the first KK mode of the gauge boson heavier. In our case, since the loop function grows abruptly for large ν, the present analysis of $B\bar{B}$ mixing can provide another hint for the upper bound of ν when the uncertainties of the parameters for ΔM_B are reduced, just as in [13].

One thing to be noticed is that $B\bar{B}$ mixing involves only the up-type quark's mass parameter ν. The main motivations of KKS are
(i) all the fermions can reside in the bulk;
(ii) different mass parameters for the top and bottom sector can accommodate the electroweak precision data.

From this point of view, $D\bar{D}$ is a good testing ground for the bottom sector. A preliminary result of our approach to $D\bar{D}$ mixing is that the contribution of the first few KK modes is extremely small. This is mainly due to the smallness of the b quark mass. Consequently, mixing with the KK masses is quite weak and the GIM cancellation is more complete. Unfortunately, the current experimental results are rather poor [16]. A more reliable check on the model can be implemented by a simultaneous analysis of $B\bar{B}$ and $D\bar{D}$ with improved measurements, as well as of the electroweak precision tests.

6 Summary

In this paper, we have calculated the $B\bar{B}$ mixing in the RS-bulk model. The contributions from the newly introduced KK modes are positive and increase the loop function. We found that KKS predicts a smaller contribution than HPR due to the remnant of the GIM cancellation. A more precise determination of the CKM unitarity triangle will test the validity of the model. Also, further developments of the present work, such as including much more KK modes or the simultaneous application to various observables, remain challenging.

Acknowledgements. This work was supported by the BK21 Program of the Korean Ministry of Education.

References

1. N. Arkani-Hamed, S. Dimopoulos, G. Dvali, Phys. Lett. B **429**, 263 (1998); Phys. Rev. D **59**, 086004 (1999); I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos, G. Dvali, Phys. Lett. B **436**, 257 (1998)
2. L. Randall, R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999); **83**, 4690 (1999)
3. T. Appelquist, H.-C. Cheng, B.A. Dobrescu, Phys. Rev. D **64**, 035002 (2001)
4. W.D. Goldberger, M.B. Wise, Phys. Rev. D **60**, 107505 (1999); Phys. Rev. Lett. **83**, 4922 (1999)
5. H. Davoudiasl, J.L. Hewett, T.G. Rizzo, Phys. Lett. B **473**, 43 (2000); A. Pomarol, Phys. Lett. B **486**, 153 (2000)
6. S. Chang, J. Hisano, H. Nakano, N. Okada, M. Yamaguchi, Phys. Rev. D **62**, 084025 (2000)
7. Y. Grossman, M. Neubert, Phys. Lett. B **474**, 361 (2000)
8. T. Gherghetta, A. Pomarol, Nucl. Phys. B **586**, 141 (2000)
9. H. Davoudiasl, J.L. Hewett, T.G. Rizzo, Phys. Rev. D **63**, 075004 (2001)
10. J.L. Hewett, F.J. Petriello, T.G. Rizzo, J. High Energy Phys. **09**, 030 (2002)
11. C.S. Kim, J.D. Kim, Jeonghyeon Song, Phys. Rev. D **67**, 015001 (2003)
12. Heavy Flavor Average Group; http://www.slac.stanford.edu/xorg/hfag/index.html
13. D. Chakraverty, K. Huitu, A. Kundu, Phys. Lett. B **558**, 173 (2003)
14. A.J. Buras, M. Spranger, A. Weiler, Nucl. Phys. B **660**, 225 (2003)
15. L. Lellouch, Nucl. Phys. Proc. Suppl. **117**, 127 (2003).
16. R. Godang et al., CLEO Collaboration, Phys. Rev. Lett. **84**, 5038 (2000); G. Raz, Phys. Rev. D **66**, 057502 (2002)